We would like to prove a periodic solution for the following equation

$$x'' = f(t, x, x')$$

if f is ω-periodic with respect to t, that is,

$$f(t+\omega, x, x') = f(t, x, x').$$

We first construct a Green function $G(t, s)$ for the following equation

$$\begin{cases} x'' + \lambda x = \delta(t-s) \\ x(0) = x(\omega) \\ x'(0) = x'(\omega) \end{cases}.$$

1- The study of Green function, and constructing the appropriate Green function for the above equation.

After that, the given differential equation is reduced to the following integral equation

$$x(t) = \int_0^{\omega} G(t, s)\{f(s, x(s), x'(s)) - \lambda x(s)\}\, ds.$$

The the existence of a periodic solution to the given differential equation is reduced to the existence of a solution for the obtained integral equation.

After that we need some fixed point theorem that goes beyond of any undergraduate course. If you would like to get familiar, I can definitely help.